From these available data on jet velocities, it appears that Equation (3) is a good and acceptable relation, where K has the value 1.41.

It is of interest to return to Equation (1) as derived by Abramovich (1963). Equation (3) differs from Equation (1) only in that $K N_{Re}^{0.135}$ has been replaced $(0.48/a)$ as shown in (4). Since Abramovich quotes a value of a equal to 0.076 for a turbulent velocity field with Reynolds numbers between 0.2×10^5 and 40×10^5, it is interesting to use Equation (4) to calculate the Reynolds number which would give the value of 0.076 for a. If we use values of 0.076 for a and 1.41 for K, the solution of the equation results in a Reynolds number of 7×10^4 which is in the range of the Abramovich figures wherein he discounted the effect of N_{Re}. Accordingly, the value of K of 1.41 used with Equation (4) is consistent with the theoretical work and experimental references used by Abramovich for air jets.

It is, therefore, recommended that the following relation be used for axial velocities for a circular jet:

$$u_r = (u_x/u_0) = 1.41\, N_{Re}^{0.135}(D_0/x) \qquad (5)$$

NOTATION

a = constant
D = diameter, m
K = constant
m = meters
N_{Re} = Reynolds number Du/ν
s = seconds
u = velocity, m/s
x = distance along axis from jet origin
ν = kinematic viscosity, m^2/s
θ = angle of expansion of jet

Subscripts

m = maximum
0 = at jet origin
x = distance along axis from jet origin
30 = 30 jet diameters from origin

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High Gradient Magnetic Separation In A Viscous Flow Field

DANIEL L. CUMMINGS,
DENNIS C. PRIEVE
and
GARY J. POWERS

Department of Chemical Engineering
Carnegie-Mellon University
Pittsburgh, Pennsylvania 15213

The basic equations governing the motion of a small paramagnetic particle in a high gradient magnetic separator (HGMS) were derived in an earlier paper (Cummings et al., 1976). The purpose of this note is to extend the previous theory by modeling the fluid flow with Lamb's solution and by including the close range London forces in the force balance.

A HGMS usually consists of a solenoidal electromagnet which has its interior packed with thin metal strands or

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Correspondence concerning this note should be addressed to Gary J. Powers.

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screens. As fluid flows through this packing, magnetic and paramagnetic particles are captured. In this development, the metal strand is modeled as an infinite cylinder and the particle as a homogeneous sphere. The geometry is defined in Figure 1.

THE FORCE BALANCE

The forces acting on the particle include inertia, magnetic forces, fluid drag and London forces:

$$\bar{F}_p = \frac{4}{3}\pi R_p{}^3\, \rho_p \frac{d\bar{v}_p}{dt} = \bar{F}_M + \bar{F}_D + \bar{F}_L \tag{1}$$

Gravity will not be considered. The magnetic force is given by

$$\bar{F}_M = (-8\pi R_p{}^3 R_c{}^2 \mu_f H_o{}^2 \alpha \Gamma) \left[\left(\frac{\alpha R_c^2}{r^5} + \frac{\cos 2\theta}{r^3}\right) \bar{i}_r + \left(\frac{\sin 2\theta}{r^3}\right) \bar{i}_\theta\right] \tag{2}$$

where

$$\alpha = \frac{\mu_c - \mu_f}{\mu_c + \mu_f} \tag{3}$$

$$\Gamma = \frac{\mu_p - \mu_f}{\mu_p + 2\mu_f} \tag{4}$$

The drag force on a small particle is given by

$$\bar{F}_D = 6\pi\eta R_p \left[\bar{i}_r \cdot \left(f_2 \bar{v}_f - \frac{\bar{v}_p}{f_1}\right) \bar{i}_r + \bar{i}_\theta \cdot \left(\frac{f_3 \bar{v}_f - \bar{v}_p}{f_4}\right) \bar{i}_\theta\right] \tag{5}$$

The universal hydrodynamic functions f_i provide a correlation to the classical Stokes drag when the particle is near the collector surface (Brenner, 1961; Goldman, Cox and Brenner, 1967*a,b;* Goren and O'Neill, 1971).

The expression for the unretarded London force is given by Spielman and Fitzpatrick (1973):

$$\bar{F}_L = -\left(\frac{2A}{3R_P}\right)\left[\frac{1}{((h/R_p) + 2)^2 (h/R_p)^2}\right] \bar{i}_r \tag{6}$$

where $$h = r - R_c - R_p = \text{Gap} \tag{7}$$

$$A = 10^{-19} J = \text{Hamaker's constant} \tag{8}$$

Substituting (2), (5), (6) into (1) and arranging in dimensionless form, one obtains

$$\left(\frac{Re_p R_p'}{9}\right) \frac{d\bar{v}_p'}{d\tau} = C_M \left[\left(\frac{\alpha}{r'^5} + \frac{\cos 2\theta}{r'^3}\right) \bar{i}_r + \left(\frac{\sin 2\theta}{r'^3}\right) \bar{i}_\theta\right] + \left[\bar{i}_r \cdot \left(f_2 \bar{v}_f' - \frac{\bar{v}_p'}{f_1}\right) \bar{i}_r + \bar{i}_\theta \cdot \left(\frac{f_3 \bar{v}_f' - \bar{v}_p'}{f_4}\right) \bar{i}_\theta\right] - \left[\frac{N_A}{(h' + 2)^2 (h')^2}\right] \bar{i}_r \tag{9}$$

where

$$r' = r/R_c \tag{10}$$

$$R_p' = R_p/R_c \tag{11}$$

$$h' = (h/R_c)/R_p' \tag{12}$$

$$\bar{v}_p' = \bar{v}_p/v_\infty \tag{13}$$

$$\bar{v}_f' = \bar{v}_f/v_\infty \tag{14}$$

$$\tau = (t\, v_\infty/R_c) \tag{15}$$

$$Re_p = (\rho_p v_\infty 2R_p/\eta) \tag{16}$$

$$Re_c = (\rho_f v_\infty 2R_c/\eta) \tag{17}$$

$$C_M = (-2/3) R_p'^2 Re_c N_M \alpha \Gamma \tag{18}$$

$$N_M = (\mu_f H_o^2)/(\rho_f v_\infty^2) \tag{19}$$

$$N_A = (A)/(9\pi\eta R_p^2 v_\infty) \tag{20}$$

The physical situation in a magnetic separator is such that typically $R_p \leq 1 \times 10^{-6} M$, $R_c \approx 50 \times 10^{-6} M$, $v_\infty < 0.1$ m/S. Under these conditions, $d\bar{v}_p'/d\tau$ is very small and may be set equal to zero (no inertia assumption). Thus, (1) can be reduced to

$$\bar{F}_p = 0 = \bar{F}_M + \bar{F}_D + \bar{F}_L \tag{21}$$

COMPUTATIONAL METHODS

The objective of this analysis is to determine the capture cross section (or deposition coefficient) Y of a single fiber. ($Y = y/R_c$, where y is the width of the free stream from which all particles will be captured.) To this end, particle trajectories were generated by use of a fourth-order Runge-Kutta method to solve (1). Similar calculations were done with a first-order (algebraic) integration of (21). With step size restrictions of $\Delta r \leq 0.02$ and $\Delta\theta \leq 1$ deg, the second method gave values of Y within 5% of the first method while reducing computation time by a factor of 100. Consequently, the results reported here were obtained from the algebraic solution of (21).

POTENTIAL FLOW

Assuming potential flow, we get

$$\bar{v}_f' = (1 - /r'^2) \cos\theta \bar{i}_r - (1 + 1/r'^2) \sin\theta \bar{i}_\theta \tag{22}$$

There are then three parameters in (21): C_M, α, N_A. Since the London forces are close range forces, the adhesion group N_A was found to affect the site of capture but not the capture cross section. This result can also be reached by an order of magnitude analysis since $N_A << C_M$, and thus for $r \geq 0.5\ R_p$, the magnetic force term in (9) dominates. However, when the particle is very near the collector surface, the London forces become dominant, and the final capture site is slightly different than if the particle were only influenced magnetically. The upper family of curves in Figures 2 shows the effects of C_M and α and, in essence, is a condensation of the results of our earlier study (Cummings et al., 1976). The plot shows that for

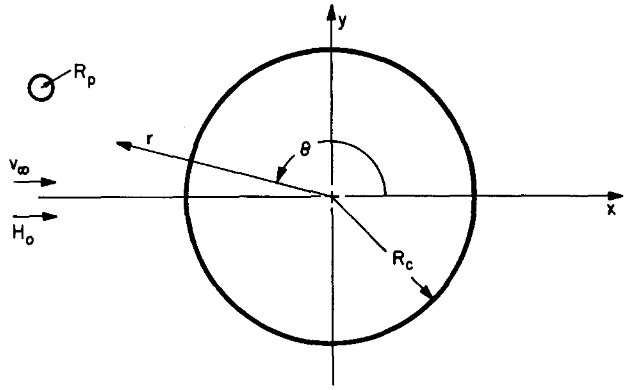

Figure 1. System geometry for capture of a small paramagnetic particle on a ferromagnetic wire. $R_p << R_c$ and the wire is infinitely long.

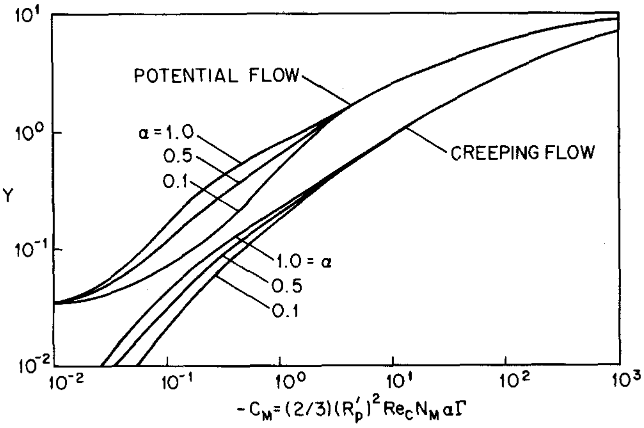

Figure 2. Capture cross section Y as a function of the coefficient of magnetic force C_M and the cylinder magnetization α. Note the large difference in Y caused by a difference in assumption about the flow field.

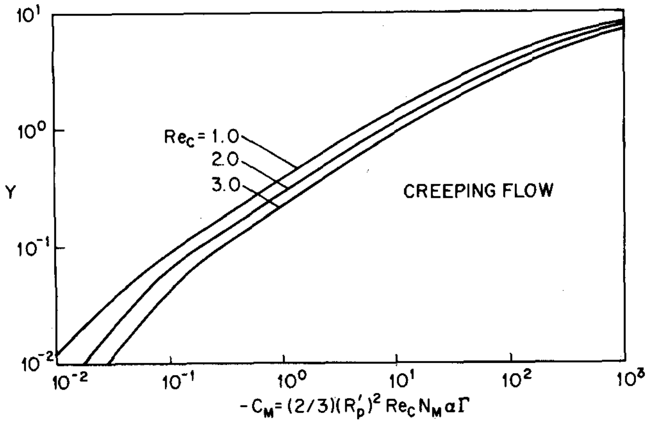

Figure 3. Effect of Re_c on Y in viscous flow. Note that with all other variables held constant, a higher Re_c has a higher capture cross section Y because $-C_M$ increases linearly with Re_c.

$C_M > 5$, α no longer affects Y, and particle capture is determined solely by the long range, $1/(r')^3$, term in (9). Watson (1975) discusses this in terms of removal of a given particle size fraction.

VISCOUS FLOW

Commercial separators operate at $Re_c < 10$, so further calculations were done assuming that the flow field around the cylinder is described by Lamb's solution (Batchelor, 1970):

$$\bar{v}_f' = C_L[\ln r' - 0.5(1 - 1/r'^2)]\cos\theta\,\bar{i}_r - C_L[\ln r' + 0.5(1 - 1/r'^2)]\sin\theta\,\bar{i}_\theta \qquad (23)$$

where

$$C_L = 1/\ln(7.4/Re_c) \qquad (24)$$

This solution was used beyond the normal range of applicability, up to $Re_c = 3$. This was justified by comparing (23) and (24) with the results of Keller and Takami (1966), generated from a numerical solution of the complete Navier-Stokes equations at $Re_c = 4$. In the region $3\pi/4 < \theta < 5\pi/4$, where particle capture predominates, Lamb's solution still provides a good description of the flow field.

There are then four parameters in (21): C_M, α, N_A, Re_c. Again, N_A affects only the point of capture and not the capture cross section. The lower family of curves in Figure 2 shows the effects of C_M and α. As before, Y depends on α in the low C_M region. The most notable feature of this plot is that the magnitude of Y is much smaller for viscous flow than for potential flow, generally by a factor of 3. The last parameter is Re_c, and its effects are shown in Figure 3. As expected, if one associates Re_c with v_∞, a lower velocity increases capture.

CONCLUSIONS

The capture cross section (deposition coefficient) for a small, paramagnetic particle approaching a bare, ferromagnetic wire has been calculated from the force balance. The cross section is a function of the coefficient of magnetic force C_M and the collector magnetization α. The slow, viscous flow field represented by Lamb's solution is physically more meaningful than potential flow since it includes the no-slip boundary condition and is more appropriate for the range of Re_c encountered in practice. The particles follow fluid streamlines which diverge much farther upstream, and the result is a large decrease in capture as compared to potential flow. The close range London forces were found to affect the site of capture but not the capture cross section.

NOTATION

A = Hamaker's constant, $10^{-19} J$
F_D = fluid drag force, N
F_L = London forces, N
F_M = *magnetic force, N*
H = magnetic field strength, (A − turns)/m
h = gap between particle and collector surfaces, m
i = unit vector
r, θ = cylindrical coordinates
R = body radius, m
t = time, s
v = velocity, m/s
η = viscosity, P_a − s
μ = magnetic permeability, H/m
ρ = density, kg/m^3

Dimensionless Groups

C_L = coefficient in Lamb's solution
C_M = coefficient of magnetic force
f_1, f_2, f_3, f_4, = universal hydrodymanic functions
N_A = adhesion group
N_M = magnetic group
Re = Reynold's number
Y = capture cross section
α = cylinder magnetization
Γ = particle magnetization
τ = time

Subscripts

c = cylinder
f = fluid
o = background value
p = particle
r = radial direction
θ = angular direction
∞ = value at infinity

Superscripts

$'$ (prime) = dimensionless value

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Sorption Kinetics of *n*-Decane on 5A Zeolites from a Nonadsorbing Liquid Solvent

J. CARO
M. BÜLOW
and
J. KÄRGER

Central Institute of Physical Chemistry
Academy of Science of the
German Democratic Republic
DDR-1199 Berlin, Rudower Chaussee 5

With the increasing application of synthetic zeolites both in catalysis and selective sorption processes, the investigation of sorption kinetics has become a subject of intensive research. In view of this, new experimental techniques, for example, the NMR pulsed field gradient technique, have been developed and applied to sorption systems (Pfeifer, 1972, 1977). The comparison of the intracrystalline self-diffusion coefficients obtained by the method mentioned above with the sorption diffusion coefficients calculated for the intracrystalline region reveals serious differences (Kärger and Caro, 1975, 1977; Ruthven, 1977; Gelbin, 1979). The results obtained by NMR pulsed field gradient technique were proved to be true in a number of experiments, whereas sorption diffusion coefficients of several *n*-paraffins on 5A zeolites with different crystal sizes show an unreasonable crystal size dependence (Kärger et al., 1976, 1977; Bülow et al., 1980).

Having excluded the rather improbable possibility that the zeolite structure is correlated with the crystal size in such a pronounced way, it must be concluded that sorption is controlled by processes different from intracrystalline diffusion. Proportionality between the sorption diffusion coefficient and the crystal size which could be observed in some experiments (Kärger et al., 1976, 1977; Bülow, 1978) would suggest that the sorption is rather affected by surface effects than influenced by intracrystalline diffusion.

As a model system which allows us to control or to avoid different factors and processes influencing uptake rate in a defined way, we have chosen the sorption of *n*-decane from a nonadsorbing liquid solvent (that is, from a solvent the molecules of which are too large to enter the zeolitic micropores). Thus it should be possible to simulate barriers of different efficiency at the outer crystal surface.

The sorption of *n*-decane from a nonadsorbing liquid solvent is characterized by the following:

1. Generally, owing to the heat of sorption, the temperature of the sorbent changes significantly during the sorption process. In liquid sorption experiments, the rapid heat transfer between the zeolite crystal and the surrounding liquid solvent ensures isothermal conditions and excludes the possibility that the heat transfer may be rate limiting.
2. Using different solvents with comparable viscosities, intercrystalline transport behavior of the sorbate molecules should be expected to be unchanged.
3. Therefore, the sorption rate of the sorbate molecules should be influenced only by the nature of mutual interactions between solvent molecules and outer crystal surface.
4. Differences in the uptake rates for different nonadsorbing solvents would give direct evidence that sorption is controlled by processes different from intracrystalline ones. In this case, the sorption has to be interpreted by assuming a surface barrier transport mechanism.

J. Kärger is at the Karl-Marx-University Leipzig, Department of Physics, NMR-Laboratory.